Mirages at Lake Geneva: the Fata Morgana

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Fata Morgana mirages are frequently seen at Lake Geneva. We show the first photographs of them, including a real-time video, and explain their main features, which are due to the very turbulent entrainment zones of capping inversions, especially in valley circulations.

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1. INTRODUCTION

The term *Fata Morgana* is widely used for vivid superior mirages showing rapidly-changing multiple images, often mistaken for architectural objects like towers [1–3], "series of pilasters" and "regular columns" [2, 4], aqueducts [5, 6], and multi-story structures such as castles [1, 2, 7] and palaces [2], monuments [7, 8], skyscrapers [9], apartment blocks [10], or pueblos [11]. These phenomena have never been explained in a fully consistent way.

The standard reference is Pernter and Exner [12], who cited one review [10, 13], but missed another [14]. These works preceded the development of boundary-layer meteorology, so their attempts to explain these mirages are obsolete today. As Forel [15] criticized the "insufficient and contradictory" reports on which [12] is based, new observations made with modern equipment must be combined with realistic atmospheric models [16, 17] to understand Fata Morganas.

The name "Fata Morgana" was first used for mirages in the Strait of Messina [5]; but they are also seen throughout southern Italy [1, 3], at Lake Geneva [8, 18], and elsewhere [7, 9, 11, 19, 20]. And, as only 15 displays were reported from the Strait of Messina from 1643 to 1903 [14], while Forel [15] saw them "four or five hundred times" in a few decades at Lake Geneva, we decided to repeat and extend his work there.

A. Nomenclature

Unfortunately, some (mostly German) writers have applied the name *Fata Morgana* to other refraction phenomena, such as looming [21, 22], and even to ordinary inferior mirages [23, 24]; Werner Herzog's 1971 film "Fata Morgana" shows only inferior mirages. But, as ten Kate [25] says, "The Fata Morgana are also mirages, but of a much more complex form . . . ." Careful students of mirages [12, 26, 27] reserve the term for displays like those shown here.

2. OLD ITALIAN OBSERVATIONS

The earliest Italian reports are from the 16th Century: "And sometimes you will see cities and castles and towers, and sheep and different colored cattle and images or specters of other things, where there is no city, no sheep, not even a thorn bush." [1] The first account of a specific display [5] included "thousands of pilasters, all equal in altitude, distance, and degree of light and shade," followed by "castles . . . , towers. . . , colonnades," etc. [6].

Minasi's classic review [2, 4] was garbled in translations: the phrase "alleys of trees" in Nicholson's version [4] became "valleys . . . " in Brewster's Edinburgh Encyclopædia [28] and many later works. (Actually, Minasi [2] himself merely said *lunghissimi alberi* ["very tall trees"].) Minasi's monograph [2] is more informative than the brief excerpt [4] considered by later writers [29] would suggest. In particular, he notes that Fata Morganas are visible during the months of late spring and summer: the usual superior-mirage season, when the water, still cold from the winter, helps produce low-level thermal inversions.

Pernter and Exner [12] reviewed the visual observations up to about a century ago, pointing out that the most reliable of the older accounts is certainly Giovene's [3, 30]. (However, they failed to correct an earlier error in translating his original report [3] from Italian to German [30]: in abbreviating compass directions, the letter O stands for *Ovest* [West] in Italian, but *Ost* [East] in German. Thus the notation SSO in [30] should have been SSW, and the SSE used by [12] must also be changed to SSW for the geographical narrative to make sense. A rough English translation of [3] is available at [31].)

3. OTHER OBSERVATIONS

Charles Dufour discovered [8], during a year's daily observations, that Fata Morgana displays occur at Lake Geneva. On

Fig. 1. A Fata Morgana mirage of Montreux, Switzerland, at 15:27:55 UT on April 22, 2015. The miraged objects are 29.1 km from the camera at the left side, and 29.4 km at the right. The striated zone contains a vertically stretched 3-image mirage of objects between 25 and 40 m above the lake; the Montreux Palace is identifiable in it, above the small boat on the left. Notice the miraged zone including this boat and seen on the lake as well. The sailboat is a Jeanneau Sun Way 21.

sunny afternoons in spring and summer, he saw objects on the shore some 30 km away, which were normally hidden by the curve of the Earth, slowly emerge from the lake horizon: "There were palaces, buildings, superb monuments, remarkable above all for their height. . . . However, their form was so strange that it was impossible for me to recognize any of the places in the countryside. On carefully taking the direction of these enlarged objects, and relating it to the federal map, I was able to convince myself that these palaces, these buildings, these new monuments were nothing but La Tour-de-Peilz [a village of about a thousand inhabitants in 1854], whose buildings, which in reality are nothing extraordinary, took on an enchanted appearance under the influence of the meteorological circumstances. . . . But objects a little more elevated above the level of the lake had quite a different appearance. Far from being enlarged vertically . . . , on the contrary they suffered a considerable depression . . . ."

Another careful visual observer was Dufour's follower Forel [15, 18, 32–34], who eventually saw the Fata Morgana over Lake Geneva "some four or five hundred times." He emphasized the formation of a "striated zone", looking like "a great city built on the shore . . . in a region known to be occupied by a few scattered huts." Forel ascribed the Fata Morgana to the transition from inferior to superior mirages, without a plausible explanation. But he recorded important geometric details: the distance to the objects seen in Fata Morganas ranged from 10 to 30 [15] or 40 km [34]; the optimal eye height was from 2 to 4 meters above the lake, and "a shift of a foot or two above or below the best position in any given case is sufficient to make the phenomenon disappear" [15]; and the width of the striated zone was 5 to 10 arcmin [34]. Refraction over cold water generally produced a concave appearance of the lake surface, whether the Fata Morgana appeared or not. He also noted that the display occupied only "some 5, 10, or 15 degrees" [18] of azimuth at a time, and appeared only on clear, calm afternoons in spring and summer.

More recent visual [9, 11] and photographic [35–40] observations share the characteristic features of earlier ones; e.g., Brooks [9] saw the Isles of Shoals (about 20 km from his position on Hampton Beach, Mass.) "like a city of skyscrapers of uniform height" from a height of 8 feet [2.4 m]. The hallmark of these mirages is their vertically elongated structure (i.e., Angelucci's colonnades [5]; Minasi's pilasters and columns; Forel's "striated zone"). This feature needs to be explained quantitatively.

4. NEW OBSERVATIONS AT LAKE GENEVA

The photographs presented throughout this paper were made by one of us (EF) at Lake Geneva, from a height of 4 m above the lake, with a Canon 5D Mark II camera body on a Takahashi FS-78 telescope with its Vari-Extender giving a nominal magnification of 2.5×. The scale has been found from images of the Pleiades to be 0.813 arcseconds per 6.4-micron pixel.

The images were recorded in RAW format, and full frame pictures are flat-field corrected to remove vignetting. Depending on the optical depth, the pictures are more or less balance corrected and the effect of the atmospheric scattering is attenuated, to better see the features flattened by the distance, with filtering techniques using local contrast enhancement in the mid-tones and selective adjustment of the levels through a mask based on image brightness.

To increase the chance of success, one must see a wide range of distances on the opposite shore. Monitoring a large panorama for small phenomena for hours is exhausting, and the risk of missing something is high. Regularly scanning the horizon with binoculars can help spot interesting events at a particular distance. In 60 hours at Lake Geneva on selected calm and sunny days in spring, something interesting was seen about 10% of the time, and exceptional displays, 2% of the time.

A. A typical Fata Morgana mirage

We begin with a view of Montreux, Switzerland, about 29 km from the camera (Fig. 1). As Forel [18] says, "The striated zone of the Fata Morgana is very remarkable and its appearance is characteristic. It is limited below by the plane of the lake which forms its lower edge; above, by a line more or less well drawn, generally horizontal. These two lines are ordinarily parallel. . . . The alternating bright and dark stripes, diversely colored, now broad, now narrow, irregular, are perfectly vertical; but their contours are indefinite, and they do not have the sharpness of real objects visible on the coast itself."

Above this zone, the coast "is strongly depressed, and the features of its landscape are altered; the vertical dimensions there are reduced as in the ordinary refractions over cold water." (See p. 544 of [18]; cf. [8].) We return to this effect in Section 6A.

B. The concave surface

Forel frequently mentions the concave appearance of the lake when the water is colder than the air, even without a Fata Morgana [18, 33]; and Brandes [41] had previously noticed that a concave appearance of the sea accompanied strong looming. The concave appearance is not obvious in Fig. 1, but can be shown by using the sailboat, whose dimensions are known (e.g., the length of the mainsail is 7.6 m). From the known scale of the image, we find that this boat is 2.43 km from the camera. Also, the apparent horizon at the lower edge of the striated zone is 7.2 arcmin above the boat's waterline.

If the lake surface were flat, the dip of the waterline would be 6.2 arcmin, as seen from the camera height of 4.4 m above the surface; and the dip of the far shore would be half a minute. So the apparent angle between the boat's waterline and the far shore would be 5.7 arcmin; but the measured angle (7.2 arcmin) is 1.5 arcmin larger: the distant shoreline appears *higher* than it would if the lake were flat. So, the lake appears to be concave.

5. EXPLAINING THE STRIATIONS

Everett [42, 43] correctly inferred that the real objects whose vertically-stretched, astigmatic images form the Fata Morgana must be optically conjugate to the observer's eye. Like both earlier [44] and later [35] workers, he assumed a parabolic refractivity profile would produce this lens-like imaging.

However, such a profile is very unrealistic. While [35] correctly point out that the power of such an atmospheric lens depends on vertical changes in the temperature gradient — i.e., on the *second* derivative of the temperature profile — this derivative must be nearly constant for several meters of height to produce the required focusing. But actual high-resolution profiles [45–47] show much fine structure at meter and centimeter scales, so that the second derivative fluctuates wildly, even changing sign over small intervals of height. This produces irregular images, such as those seen through the stable boundary layer off the Italian coast [48]; see [49] for interpretation of those photographs.

Instead of assuming a perfect atmospheric lens, we can produce good focusing with an atmospheric mirror, whose optical quality depends only on its shape. The basic idea is that a strong thermal inversion acts like a concave reflecting surface. Ray-tracing simulations [50, 51] show that Wegener's simple discontinuity model [52, 53] can produce complex mirages. And a density discontinuity was also used in early models of capping inversions [54].

A. A toy model

As a first approximation, improved in the next sections, we model an inversion as a density discontinuity that produces a quasi-reflection [42], with straight rays below the inversion. Wegener [52] shows that neglecting their curvature is equivalent to enlarging the Earth's radius; he also [53] provides formulas for calculating the geometry in detail, even when the rays below the inversion are not straight, but curved.

If the reflecting surface were elliptical in vertical section, an object at one focus of the ellipse would be imaged at the other. In the calm air of Fata Morgana displays [1–3, 32]), constant-density surfaces have the curvature of the geoid. The inversion is not elliptical; but, if it is a level surface, it differs little from a very eccentric ellipse near the minor axis.

Fig. 2 shows an inversion (the heavy circular arc ZB) and an osculating ellipse (dashed), with center at O and foci at F and F′. A ray from one focus is reflected at B to the other; rays reflected at points on the inversion near B will be very nearly imaged at the second focus as well. So the image of F′, as seen from F, is a vertical line: an element of the striated zone.

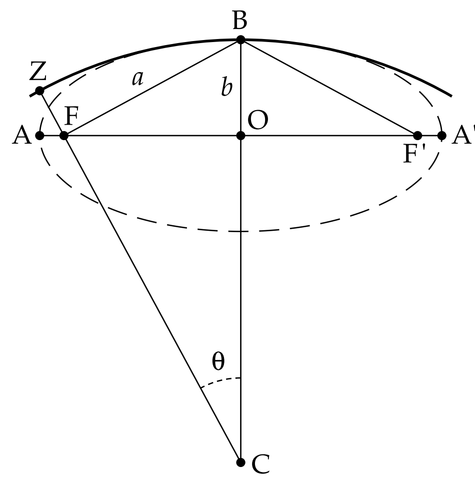

Fig. 2. Quasi-reflection at a sharp thermal inversion. FB is the astronomical horizon ray for an observer at F.

The miraged object is at the same height above the water as the eye of the observer, which agrees with Dufour's observation [8] that the miraged objects are on the far shore. Allowing for the curvature of rays beneath the inversion raises the center of the striated zone from the astronomical horizon to a small positive altitude.

B. Improving the model: valley inversions

The toy model has been checked by detailed ray-tracing (see [55–57]), which showed that it produces no striated zone at the astronomical horizon, although the image has a vertical tangent there. The part of the inversion near B in Fig. 2 that produces good focusing is seen very obliquely, so it is very foreshortened. Farther from B, the reflecting surface is less curved than the ideal ellipse; so the effective focal lengths of those parts are too large, and the image seen from F is an erect (though magnified) image of features above or below F′. We obviously require a more realistic inversion model than a discontinuity.

An inversion of finite thickness with a constant lapse rate (the negative of the vertical temperature gradient) was introduced by Deardorff [16], and refined by others [58–60]. Although the convective boundary layer on sunny days is several hundred meters thick over flat terrain, its capping inversion is forced downward in valleys [17, 61], and is strengthened by subsidence — especially over cold water. Also, the mountains surrounding the valley protect this low inversion from disruption by the zonal flow of the free atmosphere. Thus, local conditions at both

Lake Geneva and the Strait of Messina can produce low-level inversions that are both strong and thick.

Furthermore, because a valley inversion contains a turbulent mixture of the warm air above and cold air beneath the inversion, it is a layer of "highly turbulent entrainment" [16], where strong scattering of electromagnetic waves occurs [62]. This strong optical turbulence is obvious in the Fata Morgana photographs.

C. A thick-inversion model

Fig. 3 shows a thermal profile of this type. For simplicity, an isothermal surface layer has been used. The corresponding ray-trace (see Fig. 4) shows that this inversion corrects the spherical aberration of the toy model: a bundle of rays at the observer is well focused at a point some 30 km away, but at a considerably different height (40 m) than the observer (4 m). At a given range, the height of the conjugate depends on the lapse rate in the inversion: here, the lapse rate is -358.13 K/km.

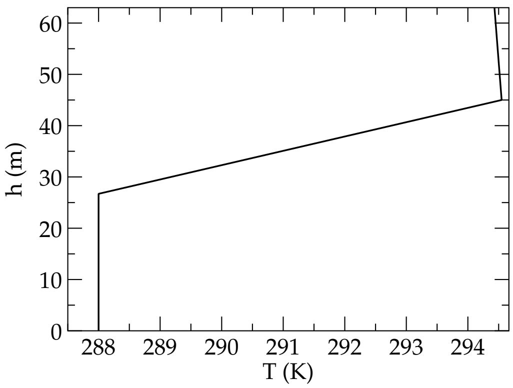

Fig. 3. Temperature profile of a thick thermal inversion.

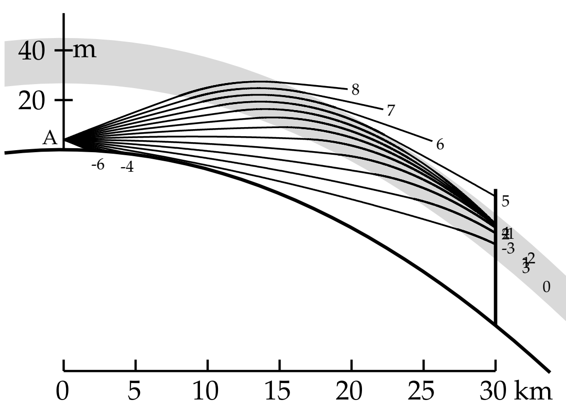

Fig. 4. Rays traced for the model of Fig. 3. The inversion layer is shaded. The observer is at A, 4 m above the lake; each ray is labeled with its altitude at the observer, in arcmin. Note the convergence of the rays, 40 m above the lake, on the vertical target 55 m high, 30 km to the right of the observer. The vertical exaggeration is about 172.

Fig. 5 shows the resulting image of a triangular target (shown undistorted in Fig. 4 of [57]); see [55] for its details. Although the target's sides slope at 45°, their images are almost vertical, forming elements of a striated zone in this partial Fata Morgana simulation. Note the compressed tip of the target, seen just above the duct. The rounded sides at the base of the image are due to assuming normal refraction beneath the inversion, which does not allow rays to be tangent to the inversion base (cf. the $-3'$ ray in Fig. 4).

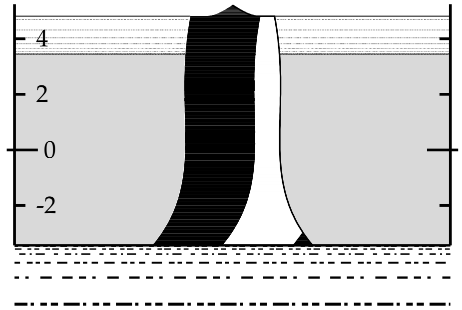

Fig. 5. Simulated image of a triangular target 30 km from an observer 4 m above the lake, in the model of Fig. 3. The vertical scale is in arcmin. The shaded zone contains rays trapped in the duct. Dashed rulings at the bottom are equally spaced on the surface; a narrow ruled zone near the top of the image contains an inverted image of the surface far behind the target (i.e., an ordinary superior mirage.)

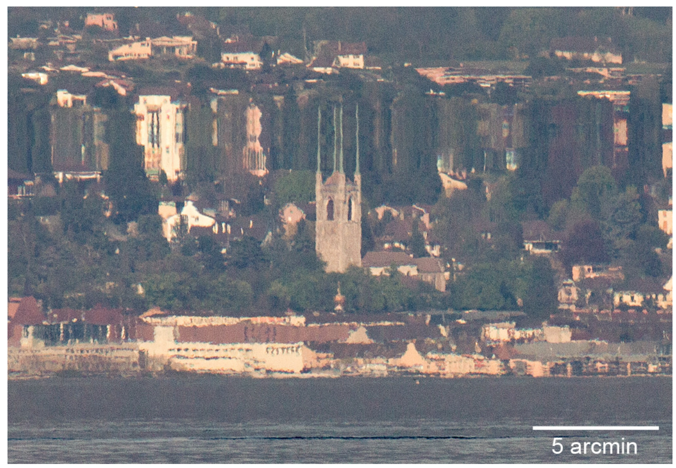

Fig. 6. A Fata Morgana image of the pinnacles on the turrets of the St. Martin church in Vevey, Switzerland, 25.2 km from the camera, at 16:06 UT on April 25, 2013; cf. [63]. The pinnacles reach 74 m above the lake. The square bell tower is 10 m on a side, and extends about 20 m above the roof of the church below it; see [64].

Fig. 6 shows a clear example of such focusing. The conical pinnacles on the turrets at the corners of the church tower extend about 3.5 m above the peak of the pyramidal roof between them. If each stretched part were an image of a whole pinnacle, it would taper to a point at the top; instead, it is almost uniformly wide. So the striations are magnified images of only a small part of the tapered pinnacles.

Identified buildings in the striated zone beyond the pinnacles lie about 1 km farther from the camera, and are not as sharply imaged. We see a mixture of erect and inverted images of those features; the background striations are much less uniform across the width of the zone than are the pinnacle images.

Fig. 6 refutes the claim by Delebecque [65] that "the objects are not enlarged" in the Fata Morgana, but that "several superimposed images of the same object . . . often encroach on one another . . . and give the illusion of an enlarged object." The

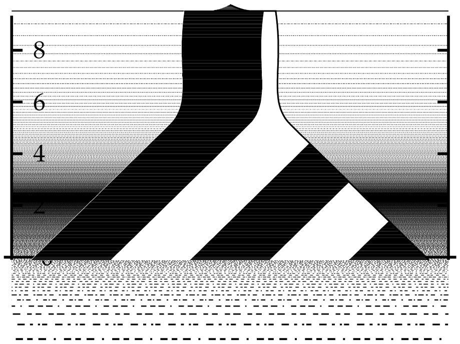

Fig. 7. Simulated image of a triangular target 58 m high, 26 km from an observer 4 m above the lake. The model lapse rate is $-140°$/km below the inversion. The vertical scale is in arcmin; cf. Figs. 5 and 6. The lower half of the target appears undistorted, while a point 40 m above the observer becomes a vertical line.

enlargement is real, not illusory.

6. RELATED PHENOMENA

A. The compressed background

Both Forel [18] and Charles Dufour [8] called attention to the contraction of images seen just above the striated zone — i.e., through the inversion. This effect is due to the divergence of rays above the top of the inversion, which maps a wide band of object space beyond the conjugate distance into a narrow zone in the image. The effect is greatest for rays that graze the top of the duct, because of the curvature of the sine law near 90°. It is well shown in Fig. 5.

B. The concave surface again

In Section 4B, we saw that the lake surface appears concave in Fig. 1. Everett [42] explained this effect with ray curvature exceeding the Earth's curvature. It requires that "the temperature must *increase* at the rate of 1 °C for 29.4 feet [8.96 m] of ascent" — i.e., a negative lapse rate stronger than about 112 °C/km.

The measured angular separation of the far shore and the sailboat's waterline in Fig. 1 requires a lapse rate near -175 °C/km in the "mixed layer", assuming it is constant. The lapse rate in the capping inversion must change by the same amount, to maintain the focusing that produces the striated zone. This adds a constant value to the density gradient everywhere below the inversion top, which is like adding an optical wedge whose thickness increases with distance. Images of vertical objects seen through a layer of constant lapse rate are shifted upward by this change, but not distorted.

However, the image is modified by the change. In Fig. 5, the base of a distant target is hidden by the lake's convex surface, which forms the lower edge of the striated zone. But now, the increased ray curvature makes objects on the far shore fully visible, so we should expect to see an undistorted image of objects below the striated zone in Fig. 1, as we do in Fig. 6. The simulation in Fig. 7 illustrates this effect.

There is now one ray at the bottom of the striated zone tangent to the inversion base; this sharpens the edge where the undistorted image (made by rays entirely below the inversion) of the base of the target meets the striated zone. Thus, the strong refraction in the lower layer produces the well-defined lower boundary of the striations, as well as the concave appearance of the lake. However, in Fig. 1, the image of objects just above the shoreline is greatly reduced in height, as will be explained in Sections 8 and 9.

Distant objects appear to be seen from an angle slightly above the horizontal, as we would expect when the lake surface appears concave. Ray bending raises the apparent horizon above the astronomical one, which accounts for Angelucci's remark [6] that the sea "swelled up . . . like a chain of dark mountains" in the display of 1643.

C. Multiple-image mirages

Fig. 8 shows a multiple-image mirage photographed about 14 minutes after Fig. 1. Because the range to the miraged objects

Fig. 8. Multiple-image mirage of the shore near Evian-les-Bains, France, at 15:41:46 UT on April 22, 2015. The distance from camera to shore was 3.31 km at the left edge, and 3.09 km at the right; the boat ramp on the left is 3.24 km away, and the part of the dock where miraged people stand is 3.12 km from the camera. The line of sight makes an angle of only 14° with the shore. The boat ramp shows 9 alternately erect and inverted images; the dock shows a 3-image mirage, becoming a 5-image mirage, from the white and red boat to the right edge. A weakly striated zone images the trees in the upper part of the picture.

Fig. 9. A comparison of the varying appearances of the Torrent area. **Top:** reference image, taken April 21, 2015, at 15:22:37 UT. **Middle:** April 22 at 15:24:21 UT; note the 5-image mirage of the building at the left, 8.0 km from the camera. The striated zone is better developed in the background, about 30 km away. As usual, objects just above it are compressed. Notice the miraged zone in the foreground, involving the lake surface, the small boat and the cape at the right edge. **Bottom:** April 22 at 15:40:16 UT. The Torrent shore is now greatly compressed. The complex striated zone in the foreground images the Torrent shore just below the building at the right; it also has erect and inverted images of the buoy at the left, the boat hull, and a person near the right edge. See Visualization 1 for a real-time video of these features between 15:35:36 and 15:37:44 UT.

is much smaller here than in Fig. 1, the striated zones are very imperfectly imaged, and consist of alternating erect and inverted images. The small refraction features in these less regular striated zones, such as the wiggles in the upper part of the boat ramp in Fig. 8, are probably caused by gravity waves on an inversion, much like the ripples seen in inferior mirages above an uneven paved road [66]. This may account for Forel's remark [32] that "the palace of the fairy Morgan only appears in all its beauty when a light breeze passes over the lake, after a morning of great calm."

The overall symmetry of the main striated zone in Fig. 8 shows how features resembling multi-story buildings appear in Fata Morgana displays.

Fig. 9 shows examples of extreme towering and stooping, as well as complex structures that accompany these Fata Morgana displays. A casual observer might take the striated band in the bottom image for a leftward extension of the vertically stretched cape in the middle image; actually, it is a vertically stretched image of the Torrent shoreline, and the cape at the right is seen almost undistorted, just beneath the striated zone. The coarse structure in the bottom image is a 3-image mirage, with small-scale erect/inverted modulations that are probably due to waves on the main inversion. Waves are well seen in the real-time video of this scene (Visualization 1).

Fig. 10. Fata Brumosa effects on April 24, 2013. The shore of Lausanne, Switzerland, 14.3 km from the camera, is shown at 15:47:49 UT on the left, and at 16:25:33 UT on the right. The coarse modulations of fine structure that slope down to the right in the striated zone are due to waves on the inversion. The white features on the left are a 3-image mirage of the boat *Helvétie* used for temporary exhibits during renovation of the Olympic Museum (light building at the left edge). In the second picture, the boat is compressed into a thin line; a second boat (brighter line at the same level on the right) is the highly compressed image of *Léman*. Note the compression of background features. The dark horizontal foreground stripes are foreshortened images of smooth patches on the lake.

An additional example of a multiple-image mirage of the kind proposed by Delebecque [65] appears in Fig. 7 of [19], where a single horizontal row of lights is stretched vertically into an illusory image of towers, each with many lights.

D. "Fog"

Many observers have noticed that Fata Morgana displays begin or end with a band of fog or haze at the horizon [14]. This illusion seems to be elicited by the indistinctness of objects seen in the striated zone (cf. Fig. 1). When the zone is too narrow for its internal structure to be made out, it may appear to be only a strip of fog.

7. CONJUGATE MIRAGES

The reversibility of light rays implies that when an observer at a point A sees multiple images of a distant point B, an observer at B sees just as many images of A. In the striated zone of a Fata Morgana, an observer at the point conjugate to an observer's eye would also see a striated zone, in which the first observer's eye would form a vertical element. However, because of the different heights of the two conjugates, the striations seen from the two conjugates will have different appearances.

In the examples presented here, the observer is clearly at the lower conjugate. However, an inversion at eye level would produce a conjugate below eye level. This may explain some of the lower striated and miraged zones projected on the lake surface, such as those in Figs. 1 and 9.

8. THE FATA BRUMOSA

Forel [18] introduced the term *Fata Brumosa* for a hazy appearance connected with the Fata Morgana. It occurs in the same conditions (warm and calm afternoons of sunny spring days), but "I find in my notes an indication of light breezes, or even local puffs of air irregularly blowing over the lake.

"The apparition consists essentially of a semblance of mists rising in places above the lake, yet these mists not having any reality and being nothing but plays of light. The lake is white, the opposite coast gray; the pseudo-mists have exactly the tint of the lake, so that in certain cases they resemble gigantic waves swelling up the surface of the lake." He offers an example of one like "a gigantic tide . . . of ten or twenty meters height that progressively covered the houses and trees of the shore." But at other times, the mists "seem to float in the air."

He says that in all cases, the apparent horizon of the lake is irregular; "instead of being a straight line, sharp and distinct, it is undulating, irregularly dented, oscillating, vibrating." [Forel ([18], p. 565) uses "vibration" to describe the optical effects of turbulence — what an astronomer would call "bad seeing".] Finally, the Fata Brumosa is "barely elevated; . . . at most 2 or 4 minutes," and can be "the elevated part of the horizon that limits the edge of the striated zone of the Fata Morgana."

Bonnelance [67] also mentions seeing the Fata Brumosa at Lake Geneva. "These fogs sometimes pass in front of a boat or the landscape, and hide it." His treatment generally follows Forel's.

Fig. 10 shows an example that fits their descriptions. The Fata Brumosa in the first image appears to hide features at the shoreline inside the striated zone. The lake-colored zone in each image shows traces of vertical striations, but they are so weak that, even in these enhanced images, they are barely visible. However, a complete Fata Brumosa, as in the second image of Fig. 10, is detectable with modern binoculars by the lack of waves and ripples, the overall uniformity of the zone being diagnostic. With good optical aid, the effect is even a bit striking.

It appears that the Fata Brumosa is a Fata Morgana striated zone imaging the lake surface. The old visual observations [18, 67] were limited to transitional phases when image motion due to waves on the inversion drew the observer's attention.

We must point out that some authors (e.g., [68–70]) have not even copied Forel's term for this correctly, but have distorted *Fata Brumosa* into the barbarism *Fata Bromosa*. This error should be discouraged.

Another paper in this issue [71] is closely related to ours: Siebren van der Werf's work on the phenomenon variously called "sea-fences" or "Hafgerðingar". Not only does the phrase

Fig. 11. Mirage due to low structure at 15:46 UT on April 22, 2015, at the Marina of Hôtel des Princes at Publier, France. The motorboat is 1.66 km from the camera; its miraged parts are from 1.8 to 2.1 m above its waterline, measured on the reference image on the left, taken at 16:17 UT. The mirage shortens the windows of the buildings at the left and right edges. The limp flag shows how calm the air was.

"giant waves" appear in its title (cf. Forel's [18] *vagues gigantesques*); the thermal and optical models used are nearly identical: the eye height in [71] is 3.5 meters, while our photographs were made 4 m above the water; [71] has visualizations of simulated gravity waves, while we have a visualization that shows real waves on inversions; etc.

The main difference between the two papers is that we have used standard methods for calculating astronomical refraction, which assumes perfectly horizontal stratification of the atmosphere; while [71] deals at length with modeling gravity waves on the inversions responsible for the mirages, thus filling in a gap in our treatment. In short, the two papers reinforce one another. Taken together, they show that Forel's *Fata Brumosa* and the Norse *Hafgerðingar* are different names for exactly the same phenomenon: a Fata Morgana of the water surface.

Ref. [71] points out several photographs of the Hafgerðingar in [72]; it also appears in Greenler's Plate 7–4 [73], where it is explicitly called a Fata Morgana. (Both Fig. 2 of [72] and Greenler's Plate [73] show images of frozen rather than liquid water.) A more recent photograph is on the cover of the November, 2009, issue of *Weather* [39].

9. REAL-WORLD COMPLICATIONS

The actual boundary layer above the lake is more complicated than the simple two-layer model we have used. We have already mentioned a second miraged zone in Fig. 1; additional images taken a few minutes before and after it show that the dark and nearly featureless zone adjacent to the far shore is also towered and striated, and so is probably a Fata Brumosa, due to a third inversion, producing the image compression just above the shoreline.

It is impossible to make exact simulations of the effects seen in the photographs, because these additional boundary-layer structures are not well understood. Fig. 11 shows a 3-image mirage associated with an inversion below the camera height. A similar mirage due to a strong inversion just a couple of meters above a lake appears in Fig. 5b of [74]. Such strong refractive effects lie outside the simple model we have used to calculate the lapse rate beneath the main inversion.

10. DISCUSSION

Pernter and Exner [12] and their followers [25, 35] supposed that nearly vertical constant-density surfaces were required to explain the complex images; but this conflicts with both optical [75–77] and aerological evidence [78, 79] that the surface tilts rarely exceed two arcmin. Large deviations from the horizontal imply large deviations from hydrostatic equilibrium, and consequently large pressure gradients, and winds — contrary to the frequent observation (e.g., [1–3, 32]) of unusually calm conditions (cf. the flag in Fig. 11). Schiele's "ellipsoidal" model [80] has the same problem.

Forel [15, 32, 34] thought that a transformation from inferior to superior mirages was involved. But modern boundary-layer meteorology [16, 54, 58–60] shows that a nearly adiabatic "mixed layer" lies between the unstable surface layer that produces the inferior mirage and the capping inversion above it; and this region must separate the superior and inferior mirages — as, indeed, Forel's Fig. 150 (on p. 550 of [18]) plainly shows, for an observation on 27 June 1879. In fact, his Fig. 151 requires some layers of the atmosphere to curve rays both toward the surface and away from it — a physical impossibility, if the air is horizontally stratified (as it must be in calm conditions). Furthermore, Fata Morganas seen by others during both morning [1, 3] and evening twilight [3], and at night [19, 20], as well as those made over ice or snow [73, 81], make inferior mirages seem irrelevant: only the inversion is required. However, our new observations confirm many valid results obtained by Forel and others over a century ago that have been neglected by modern workers.

The literature of Fata Morganas has unfortunately been fragmented, with the Italian displays isolated from those seen elsewhere, and the ones around Greenland described in different terms. The best link among these groups has been Scoresby's account [7]; on p. 27, he remarks on "the beautiful basaltic character, which it is a general property of this remarkable state of the atmosphere to produce. The apparent columns were all vertical, or nearly so, and, when slightly waved, maintained their parallelism, the curvature of the adjoining columns corresponding with each other." This is clearly what Forel [18] called a "striated zone"; cf. our Fig. 1. Other accounts ([71, 72]) of mirages near

Greenland also use the image of cliffs. But so did Woltman, near Hamburg ([82], p. 430): "Sometimes a strip of air separates the inverted image from the underlying objects; yet image and object collide more frequently, and intermix so that neither of them is recognizable, and the whole looks like a high seacoast, with many vertical stripes." In sum, these mirages are seen wherever low-lying strong inversions occur, and the differences among them are terminological rather than geographic.

Although the simple model we have used shows how the main features of these mirages are produced by valley inversions, many details will require additional understanding of the boundary layer. The low inversions near the lake surface (see Fig. 11) are not understood; and the weaker inverted lapse rate that is needed in the mixed layer to explain the concave appearance of the lake surface is not predicted by dynamical meteorological models.

Perhaps additional observations of these mirages will help provide insight into these poorly understood stable boundary layers (cf. [83]).

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